Total absorption of electromagnetic waves in ultimately thin layers

Y. Ra'di, V.S. Asadchy, S.A. Tretyakov, *Fellow, IEEE*

***Abstract*—We consider single-layer arrays of electrically small lossy bi-anisotropic particles that completely absorb electromagnetic waves at normal incidence. Required conditions for electromagnetic properties of bi-anisotropic particles have been identified in the most general case of uniaxial reciprocal and nonreciprocal particles. We consider the design possibilities offered by the particles of all four fundamental classes of bi-anisotropic inclusions: reciprocal chiral and omega particles and nonreciprocal Tellegen and moving particles. We also study the reflection/transmission properties of asymmetric structures with different properties when illuminated from the opposite sides of the sheet. It has been found that it is possible to realize single-layer grids which exhibit the total absorption property when illuminated from one side but are totally transparent when illuminated from the other side (an ultimately thin isolator). Other possible properties are co-polarized or twist polarized reflection from the side opposite to the absorbing one. Finally, we discuss possible approaches to practical realization of particles with the properties required for single-layer perfect absorbers and other proposed devices.**

***Index Terms*—Electromagnetic wave absorption, absorber, periodical structures, isolator, twist-polarizer, bi-anisotropic particle, reflection, transmission, resonance**

I. INTRODUCTION

IN this paper we study possible approaches to the design of electrically thin layers (sheets) which would behave as *perfect absorbers* for normally-incident electromagnetic plane waves. We say that absorption in a layer at some frequency is "perfect" or "total", if all incident power is dissipated in the layer. This implies that both reflection and transmission coefficients are equal to zero. In this study we will consider only the case of normal incidence, thus, this term should not be confused with the *perfectly matched layer* or PML, which implies zero reflection coefficient at any incident angle and for any polarization of the incident wave.

The theory and design of absorbers for electromagnetic radiation has a long history and there exists a large variety of designs, especially for microwave frequencies (see, e.g. [1]–[3]). However, in most of these designs the absorbing structure is backed by a reflecting wall (usually a metal surface), because most often the goal is to reduce microwave reflections from metal structures. Recently, there has been considerable interest in thin absorbing layers for situations where there is no reflector behind, so that the electromagnetic properties of the object which one wants to "hide" can be arbitrary. Naturally, a thin reflector can be incorporated in the absorber structure, but often it is desirable to allow off-band electromagnetic waves to pass through the structure or avoiding conductors is one of the application requirements. Also, for infrared and visible-light applications the use of perfect reflectors as parts of absorbing layers is not practically possible except if the use of electrically thick layers of photonic crystals is allowed in design. Electrically thin matched absorbers can be realized in many ways, but, to the best of our knowledge, only a very limited set of opportunities has been explored so far. One known possibility is to combine two thin metamaterial layers with contrasting material parameters [4] or combine a thin resistive sheet with an array of small resonant split rings (which realize the necessary magnetic response) [5]. Here we will not consider such two- or multilayer structures, concentrating on the basic and fundamentally simplest case of a single sheet with properly designed properties. These single-layer absorbers provide ultimately thin design solutions, because the layer thickness cannot be made smaller than just one layer of particles (molecules).

Conceptually, the simplest possible thin absorbing sheet is a uniform or composite layer of electrically negligible thickness (impedance sheet). In this case, the incident electric field induces an infinitesimally thin sheet of electric current in the layer, which eventually leads to dissipation of the incident power, if the layer is lossy. However, it is obvious that in this case the absorbed power can reach only one half of the incident power, and the total absorption is not possible (e.g., [6]). This follows from the fact that the induced current sheet symmetrically radiates plane waves in the forward and back directions. Zero transmission coefficient implies that the amplitude of this secondary wave behind the sheet equals to that of the incident wave (so that the two waves cancel each other behind the sheet), but this means that the reflection coefficient equals unity in the amplitude. Thus, in order to enable total absorption, we need to allow also magnetic current to be induced in the layer. Strictly speaking, this implies that the layer thickness cannot be negligibly small (electrically), at least if no natural magnetics are used, but it can be still made very small compared with the wavelength.

In view of practical requirements in realization of layers with desired electromagnetic response, calling for the use of composite structures, we do not model the layer as a homogeneous sheet described by some surface susceptibility or impedance, but assume from the beginning that the layer is a composite structure made up of small polarizable particles. Engineering these inclusions, we can tune the reflection and transmission responses of the composite layer. Such artificial sheets with engineered electromagnetic properties are called

Y. Ra'di, V.S. Asadchy, S.A. Tretyakov are with the Department of Radio Science and Engineering/SMARAD Center of Excellence, Aalto University, P. O. Box 13000, FI-00076 AALTO, Finland. Email: younes.radi@aalto.fi.

"metasurfaces" or "metasheets", see a recent review in [7]. As the goal is to study possible realizations of ultimately thin perfect absorbers, we will consider single-layer arrays of dipole particles. The absorber designs which we will develop here will give the required polarizabilities of individual inclusions together with the appropriate array period.

The condition for total absorption of normally incident plane waves by a single infinite periodic array of electric and magnetic dipoles is known from the antenna theory [8]. Let us assume that an infinite array with the period a (a is smaller than the wavelength in the surrounding space) in each unit cell contains one isotropic electrically polarizable particle and one isotropic magnetically polarizable particle. Incident electric and magnetic fields will induce electric dipole moment $\mathbf{p}$ and magnetic moment $\mathbf{m}$. The two moments will be orthogonal: electric moment along the incident electric field and magnetic moment along the magnetic field. Arrays of both moments will create secondary plane waves, and in the forward direction the secondary electric field amplitude reads

$$E_{\rm forward} = \frac{-j\omega}{2S}\left(\eta_0 p + \frac{1}{\eta_0} m\right) \tag{1}$$

where $S = a^2$ is the unit-cell area, so that $j\omega p/S$ is the surface-averaged electric current density and $j\omega m/S$ is the magnetic current surface density. $\eta_0 = \sqrt{\mu_0/\epsilon_0}$ is the wave impedance of the surrounding space. Derivation of these formulas for plane-wave fields created by planar sheets of electric and magnetic currents can be found e.g. in [9]. In the opposite direction (the reflection direction), the same induced electric and magnetic currents generate a plane wave with the amplitude

$$E_{\rm back} = \frac{-j\omega}{2S}\left(\eta_0 p - \frac{1}{\eta_0} m\right) \tag{2}$$

While the secondary electric fields created by the electric current sheet are the same on both sides of the sheet, the electric fields created by the magnetic current sheet differ by sign. Now, we see that it is possible to choose the moments so that the secondary field would cancel the incident field in the forward direction (zero transmission coefficient) and at the same time the secondary field would be zero in the back direction (zero reflection coefficient). Obviously, the conditions are

$$\eta_0 p = \frac{1}{\eta_0} m \tag{3}$$

which ensures zero reflection coefficient, and

$$\frac{-j\omega}{2S}\left(\eta_0 p + \frac{1}{\eta_0} m\right) = -E_{\rm inc} \tag{4}$$

($E_{\rm inc}$ is the amplitude of the incident electric field), which ensures zero transmission coefficient. The solution of (3) and (4) reads

$$p = \frac{-jS}{\omega\eta_0}E_{\rm inc}, \qquad m = \eta_0^2 p \tag{5}$$

This arrangement of electric and magnetic current sheets is in fact a Huygens surface, and for volumetric material layers that would correspond to materials with equal relative permittivity and permeability. We note in passing that the use of volumetric materials with matched wave impedance in absorbers is well known, see e.g. [1] or [10, ch. 12]. In paper [11], it was found that such materials are the most advantageous in terms of minimizing reflections and transmission for isotropic layers of a given thickness.

Thus, a simple approach to realization of total absorbing layers is to arrange electrically and magnetically polarizable particles in a dense lattice and tune the polarizabilities so that conditions (3) and (4) are satisfied. However, this is not the only possible approach. We only need to ensure that (3) and (4) hold, but the particles in which these dipole moments are induced can be any electrically small objects which one can describe as dipole scatterers. We expect that there should be considerable design freedom and possibilities for realizing additional practically useful properties if we do not restrict the design space by the simplest case of small electrically and magnetically polarizable scatterers (like small magnetodielectric spheres, for example).

In this paper we consider planar layers formed by electrically small particles modeled by the most general linear relations between the induced dipole moments $\mathbf{p}$ and $\mathbf{m}$ and the local fields $\mathbf{E}_{\rm loc}$ and $\mathbf{H}_{\rm loc}$ at the positions of the particles:

$$\begin{bmatrix} \mathbf{p} \\ \mathbf{m} \end{bmatrix} = \begin{bmatrix} \overline{\overline{\alpha}}_{ee} & \overline{\overline{\alpha}}_{em} \\ \overline{\overline{\alpha}}_{me} & \overline{\overline{\alpha}}_{mm} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{E}_{\rm loc} \\ \mathbf{H}_{\rm loc} \end{bmatrix}. \tag{6}$$

Although for the desired operation of the layer the induced dipole moments must satisfy the same conditions of the Huygens sheet (3) and (4), the actual design space is vastly larger, since we can exploit the magneto-electric coupling parameters $\overline{\overline{\alpha}}_{em}$ and $\overline{\overline{\alpha}}_{me}$ to bring the induced moments to the desired balance and required amplitudes. Furthermore, additional functionalities will become possible, as we will see in the following.

While the simple and well-known solution in form of electric and magnetic dipole particles corresponds to simple magnetodielectric layers with $\epsilon_r = \mu_r$ (if we think about layers of homogeneous materials), the general case of bi-anisotropic polarizabilities of individual particles corresponds to bi-anisotropic absorbing layers. In the past, chiral absorbing layers were studied in detail, as it was expected that the extra material parameter, the chirality parameter, would offer more flexibility in realizing the zero-reflection condition [12]–[18], but only for metal-backed volumetric layers. The use of omega coupling phenomenon for matched absorber layers was explored in [10], [19], [20], but also only for material layers on perfectly conducting surfaces. Recently, different kinds of absorbers have been proposed to absorb electromagnetic waves in microwave or optical spectra [21]–[27]. As it was mentioned, most of these absorbers are backed with a metal sheet which limits their functionality for the wave coming from the other side. These absorbers contain more than one layer of particles and they are designed so that to absorb the wave from one side while they have some uncontrollable properties for the wave coming from the other side. Here, we answer important questions: How one can realize single-layer perfect absorbers from one side of the sheet and what functionalities can be realized for waves coming from the other side? Of course, this implies that there is no metal (PEC) ground plane

as a part of the absorbing structure.

Here, we study the possible use of single arrays of bi-anisotropic particles in free space and, in addition, consider the most general uniaxial particles and study the possible use of nonreciprocal magnetoelectric effects in the particles. This brings the number of considered distinct classes of bi-anisotropic particles to four: reciprocal chiral and omega and nonreciprocal Tellegen and "moving" particles [10], [28]. Also in this paper we consider the use of particles which have hybrid electromagnetic properties of several classes, e.g. "moving" chiral and Tellegen omega particles.

II. Total absorption in arrays of general bi-anisotropic particles

A. Effective polarizability dyadics of particles in periodic arrays

In this paper, we consider thin absorbers for normally incident plane waves and concentrate on uniaxial structures, isotropic in the plane of the layer. This property ensures that the absorber functions for arbitrary polarized incident plane waves. The orientation of the absorbing sheet in space is defined by the unit vector $\mathbf{z}_0$, orthogonal to its plane. The layer consists of an array of electrically small uniaxial particles. As discussed above, total absorption requires at least electric and magnetic dipole moments induced in the particles, and the requirement of ultimately small thickness means that higher-order multipoles are negligible. Thus, we assume that the particles are bi-anisotropic particles characterized by four dyadic polarizabilities: electric, magnetic, electromagnetic, and magnetoelectric, which relate local electromagnetic fields to the induced electric and magnetic dipole moments as in (6).

The uniaxial symmetry allows only isotopic response and rotation around the axis $\mathbf{z}_0$. Thus, all the polarizabilities in (6) take the forms:

$$\begin{array}{ll}\overline{\overline{\alpha}}_{ee}=\alpha_{ee}^{co}\overline{\overline{I}}_t+\alpha_{ee}^{cr}\overline{\overline{J}}_t, & \overline{\overline{\alpha}}_{mm}=\alpha_{mm}^{co}\overline{\overline{I}}_t+\alpha_{mm}^{cr}\overline{\overline{J}}_t \\ \overline{\overline{\alpha}}_{em}=\alpha_{em}^{co}\overline{\overline{I}}_t+\alpha_{em}^{cr}\overline{\overline{J}}_t, & \overline{\overline{\alpha}}_{me}=\alpha_{me}^{co}\overline{\overline{I}}_t+\alpha_{me}^{cr}\overline{\overline{J}}_t,\end{array} \tag{7}$$

where indices co and cr refer to the symmetric and anti-symmetric parts of the corresponding dyadics, respectively. $\overline{\overline{I}}_t=\overline{\overline{I}}-\mathbf{z}_0\mathbf{z}_0$ is the transverse unit dyadic and $\overline{\overline{J}}_t=\mathbf{z}_0\times\overline{\overline{I}}_t$ is the vector-product operator. The particles are arranged in a square lattice with the unit cell of the size $a\times a$. The grid is excited by an arbitrary polarized plane wave with the electric and magnetic fields of $\mathbf{E}_{\rm inc}$ and $\mathbf{H}_{\rm inc}$, respectively, which are uniform in the array plane (normal incidence). In this situation, the induced dipole moments are the same for all particles. We assume that the grid period a is smaller than the wavelength, so that no grating lobes are generated.

The local fields exciting the particles are the sums of the external incident field and the interaction field caused by the induced dipole moments in other particles:

$$\begin{aligned}\mathbf{E}_{\rm loc}&=\mathbf{E}_{\rm inc}+\overline{\overline{\beta}}_e\cdot\mathbf{p}\\ \mathbf{H}_{\rm loc}&=\mathbf{H}_{\rm inc}+\overline{\overline{\beta}}_m\cdot\mathbf{m},\end{aligned} \tag{8}$$

where $\overline{\overline{\beta}}_e$ and $\overline{\overline{\beta}}_m$ are the interaction constants. These dyadic coefficients are proportional to the two-dimensional unit dyadic $\overline{\overline{I}}_t$. Explicit analytical expressions for the interaction constants can be found in [10]:

$$\begin{aligned}\overline{\overline{\beta}}_e=&-\left[{\rm Re}\left\{\frac{j\omega\eta_0}{4S}\left(1-\frac{1}{jkR}\right)e^{-jkR}\right\}\right.\\ &\left.+j\left(\frac{k^3}{6\pi\epsilon_0}-\frac{\eta_0\omega}{2S}\right)\right]\overline{\overline{I}}_t\\ \overline{\overline{\beta}}_m=&\frac{\overline{\overline{\beta}}_e}{\eta_0^2},\qquad R=\frac{a}{1.438}\end{aligned} \tag{9}$$

($S=a^2$ is the area of each unit cell and k is the free-space wave number). Here the expression for the imaginary part is exact, and that for the real part is an approximation valid for $ka<1$.

Equations (6), (8), and (9) can be re-written as relations between the induced dipole moments and the incident fields:

$$\begin{bmatrix}\mathbf{p}\\ \mathbf{m}\end{bmatrix}=\begin{bmatrix}\overline{\overline{\widehat{\alpha}}}_{ee} & \overline{\overline{\widehat{\alpha}}}_{em}\\ \overline{\overline{\widehat{\alpha}}}_{me} & \overline{\overline{\widehat{\alpha}}}_{mm}\end{bmatrix}\cdot\begin{bmatrix}\mathbf{E}_{\rm inc}\\ \mathbf{H}_{\rm inc}\end{bmatrix}, \tag{10}$$

where the effective polarizabilities (marked by hats) include the effects of particle interactions in the array. Explicit formulas for the effective polarizabilities in terms of the individual polarizabilities and interaction constants read [29]

$$\begin{aligned}\overline{\overline{\widehat{\alpha}}}_{ee}=&\left(\overline{\overline{I}}_t-\overline{\overline{\alpha}}_{ee}\cdot\overline{\overline{\beta}}_e-\overline{\overline{\alpha}}_{em}\cdot\overline{\overline{\beta}}_m\cdot(\overline{\overline{I}}_t-\overline{\overline{\alpha}}_{mm}\cdot\overline{\overline{\beta}}_m)^{-1}\cdot\overline{\overline{\alpha}}_{me}\cdot\overline{\overline{\beta}}_e\right)^{-1}\\ &\cdot\left(\overline{\overline{\alpha}}_{ee}+\overline{\overline{\alpha}}_{em}\cdot\overline{\overline{\beta}}_m\cdot(\overline{\overline{I}}_t-\overline{\overline{\alpha}}_{mm}\cdot\overline{\overline{\beta}}_m)^{-1}\cdot\overline{\overline{\alpha}}_{me}\right)\\ \overline{\overline{\widehat{\alpha}}}_{em}=&\left(\overline{\overline{I}}_t-\overline{\overline{\alpha}}_{ee}\cdot\overline{\overline{\beta}}_e-\overline{\overline{\alpha}}_{em}\cdot\overline{\overline{\beta}}_m\cdot(\overline{\overline{I}}_t-\overline{\overline{\alpha}}_{mm}\cdot\overline{\overline{\beta}}_m)^{-1}\cdot\overline{\overline{\alpha}}_{me}\cdot\overline{\overline{\beta}}_e\right)^{-1}\\ &\cdot\left(\overline{\overline{\alpha}}_{em}+\overline{\overline{\alpha}}_{em}\cdot\overline{\overline{\beta}}_m\cdot(\overline{\overline{I}}_t-\overline{\overline{\alpha}}_{mm}\cdot\overline{\overline{\beta}}_m)^{-1}\cdot\overline{\overline{\alpha}}_{mm}\right)\\ \overline{\overline{\widehat{\alpha}}}_{me}=&\left(\overline{\overline{I}}_t-\overline{\overline{\alpha}}_{me}\cdot\overline{\overline{\beta}}_e\cdot(\overline{\overline{I}}_t-\overline{\overline{\alpha}}_{ee}\cdot\overline{\overline{\beta}}_e)^{-1}\cdot\overline{\overline{\alpha}}_{em}\cdot\overline{\overline{\beta}}_m-\overline{\overline{\alpha}}_{mm}\cdot\overline{\overline{\beta}}_m\right)^{-1}\\ &\cdot\left(\overline{\overline{\alpha}}_{me}+\overline{\overline{\alpha}}_{me}\cdot\overline{\overline{\beta}}_e\cdot(\overline{\overline{I}}_t-\overline{\overline{\alpha}}_{ee}\cdot\overline{\overline{\beta}}_e)^{-1}\cdot\overline{\overline{\alpha}}_{ee}\right)\\ \overline{\overline{\widehat{\alpha}}}_{mm}=&\left(\overline{\overline{I}}_t-\overline{\overline{\alpha}}_{me}\cdot\overline{\overline{\beta}}_e\cdot(\overline{\overline{I}}_t-\overline{\overline{\alpha}}_{ee}\cdot\overline{\overline{\beta}}_e)^{-1}\cdot\overline{\overline{\alpha}}_{em}\cdot\overline{\overline{\beta}}_m-\overline{\overline{\alpha}}_{mm}\cdot\overline{\overline{\beta}}_m\right)^{-1}\\ &\cdot\left(\overline{\overline{\alpha}}_{mm}+\overline{\overline{\alpha}}_{me}\cdot\overline{\overline{\beta}}_e\cdot(\overline{\overline{I}}_t-\overline{\overline{\alpha}}_{ee}\cdot\overline{\overline{\beta}}_e)^{-1}\cdot\overline{\overline{\alpha}}_{em}\right).\end{aligned} \tag{11}$$

Because the interaction constants are diagonal dyadics, the symmetry properties of the effective polarizabilities are the same as for the individual particle polarizabilities (as defined in (7)):

$$\begin{array}{ll}\overline{\overline{\widehat{\alpha}}}_{ee}=\widehat{\alpha}_{ee}^{co}\overline{\overline{I}}_t+\widehat{\alpha}_{ee}^{cr}\overline{\overline{J}}_t, & \overline{\overline{\widehat{\alpha}}}_{mm}=\widehat{\alpha}_{mm}^{co}\overline{\overline{I}}_t+\widehat{\alpha}_{mm}^{cr}\overline{\overline{J}}_t\\ \overline{\overline{\widehat{\alpha}}}_{em}=\widehat{\alpha}_{em}^{co}\overline{\overline{I}}_t+\widehat{\alpha}_{em}^{cr}\overline{\overline{J}}_t, & \overline{\overline{\widehat{\alpha}}}_{me}=\widehat{\alpha}_{me}^{co}\overline{\overline{I}}_t+\widehat{\alpha}_{me}^{cr}\overline{\overline{J}}_t.\end{array} \tag{12}$$

This can be checked by substituting (7) in (11).

B. Reflection and transmission coefficients

In the theory of absorbing sheets, we will distinguish between illuminations of the sheet from its two opposite sides, along $-\mathbf{z}_0$ and $\mathbf{z}_0$. In the rest of the paper, we will use double signs for these two cases, where the top and bottom signs

correspond to the incident plane wave propagating in $-\mathbf{z}_0$ and $\mathbf{z}_0$ directions, respectively.

In the incident plane wave, the electric and magnetic fields satisfy

$$\mathbf{H}_{\rm inc} = \mp\frac{1}{\eta_0}\overline{\overline{J}}_t \cdot \mathbf{E}_{\rm inc}. \tag{13}$$

Thus, the dipole moments in (10) can be written as

$$\begin{bmatrix} \mathbf{p} \\ \mathbf{m} \end{bmatrix} = \begin{bmatrix} \overline{\overline{\widehat{\alpha}}}_{ee} \mp \dfrac{1}{\eta_0}\overline{\overline{\widehat{\alpha}}}_{em} \cdot (\mathbf{z}_0 \times \overline{\overline{I}}_t) \\ \overline{\overline{\widehat{\alpha}}}_{me} \mp \dfrac{1}{\eta_0}\overline{\overline{\widehat{\alpha}}}_{mm} \cdot (\mathbf{z}_0 \times \overline{\overline{I}}_t) \end{bmatrix} \cdot \ \mathbf{E}_{\rm inc} \ . \tag{14}$$

Secondary plane waves (reflected and transmitted) are generated by surface-averaged current densities

$$\mathbf{J}_e = \frac{j\omega}{S}\mathbf{p}, \qquad \mathbf{J}_m = \frac{j\omega}{S}\mathbf{m}. \tag{15}$$

Radiation from infinite sheets of electric and magnetic currents can be easily solved [29] from the Maxwell equations:

$$\begin{aligned} \mathbf{E}_r &= -\frac{j\omega}{2S}[\eta_0\mathbf{p} \mp \mathbf{z}_0 \times \mathbf{m}] \\ &= -\frac{j\omega}{2S}\left[\eta_0\overline{\overline{\widehat{\alpha}}}_{ee} \mp \overline{\overline{\widehat{\alpha}}}_{em} \times \mathbf{z}_0 \mp \mathbf{z}_0 \times \overline{\overline{\widehat{\alpha}}}_{me} + \frac{1}{\eta_0}\mathbf{z}_0 \times (\overline{\overline{\widehat{\alpha}}}_{mm} \times \mathbf{z}_0)\right] \cdot \mathbf{E}_{\rm inc} \end{aligned} \tag{16}$$

$$\begin{aligned} \mathbf{E}_t &= \mathbf{E}_{\rm inc} - \frac{j\omega}{2S}[\eta_0\mathbf{p} \pm \mathbf{z}_0 \times \mathbf{m}] \\ &= \Bigg\{\overline{\overline{I}}_t - \frac{j\omega}{2S}\Big[\eta_0\overline{\overline{\widehat{\alpha}}}_{ee} \mp \overline{\overline{\widehat{\alpha}}}_{em} \times \mathbf{z}_0 \pm \mathbf{z}_0 \times \overline{\overline{\widehat{\alpha}}}_{me} \\ &\qquad - \frac{1}{\eta_0}\mathbf{z}_0 \times (\overline{\overline{\widehat{\alpha}}}_{mm} \times \mathbf{z}_0)\Big]\Bigg\} \cdot \mathbf{E}_{\rm inc}. \end{aligned} \tag{17}$$

After substituting of (12), we find:

$$\begin{aligned} \mathbf{E}_r = -\frac{j\omega}{2S}\Bigg\{&\left[\eta_0\widehat{\alpha}_{ee}^{co} \pm \widehat{\alpha}_{em}^{cr} \pm \widehat{\alpha}_{me}^{cr} - \frac{1}{\eta_0}\widehat{\alpha}_{mm}^{co}\right]\overline{\overline{I}}_t \\ &+ \left[\eta_0\widehat{\alpha}_{ee}^{cr} \mp \widehat{\alpha}_{em}^{co} \mp \widehat{\alpha}_{me}^{co} - \frac{1}{\eta_0}\widehat{\alpha}_{mm}^{cr}\right]\overline{\overline{J}}_t\Bigg\} \cdot \mathbf{E}_{\rm inc} \end{aligned} \tag{18}$$

$$\begin{aligned} \mathbf{E}_t = \Bigg\{&\left[1 - \frac{j\omega}{2S}\left(\eta_0\widehat{\alpha}_{ee}^{co} \pm \widehat{\alpha}_{em}^{cr} \mp \widehat{\alpha}_{me}^{cr} + \frac{1}{\eta_0}\widehat{\alpha}_{mm}^{co}\right)\right]\overline{\overline{I}}_t \\ &- \frac{j\omega}{2S}\left[\eta_0\widehat{\alpha}_{ee}^{cr} \mp \widehat{\alpha}_{em}^{co} \pm \widehat{\alpha}_{me}^{co} + \frac{1}{\eta_0}\widehat{\alpha}_{mm}^{cr}\right]\overline{\overline{J}}_t\Bigg\} \cdot \mathbf{E}_{\rm inc}. \end{aligned} \tag{19}$$

Using these general expressions for the reflected and transmitted fields from general bi-anisotropic planar arrays, we are ready to study how we can make these fields equal to zero, as required for perfect absorbers.

C. General conditions for total absorption

1) Total absorption from both sides of the sheet: The definition of a perfect absorber implies that

$$\mathbf{E}_r = 0, \qquad \mathbf{E}_t = 0 \ . \tag{20}$$

Equating to zero the expressions in square brackets in (18) and (19), we arrive to sufficient conditions for total absorption of arbitrarily polarized incident plane waves:

$$\begin{aligned} &\eta_0\widehat{\alpha}_{ee}^{co} \pm \widehat{\alpha}_{em}^{cr} \pm \widehat{\alpha}_{me}^{cr} - \frac{1}{\eta_0}\widehat{\alpha}_{mm}^{co} = 0 \\ &\eta_0\widehat{\alpha}_{ee}^{cr} \mp \widehat{\alpha}_{em}^{co} \mp \widehat{\alpha}_{me}^{co} - \frac{1}{\eta_0}\widehat{\alpha}_{mm}^{cr} = 0 \\ &\eta_0\widehat{\alpha}_{ee}^{co} \pm \widehat{\alpha}_{em}^{cr} \mp \widehat{\alpha}_{me}^{cr} + \frac{1}{\eta_0}\widehat{\alpha}_{mm}^{co} = \frac{2S}{j\omega} \\ &\eta_0\widehat{\alpha}_{ee}^{cr} \mp \widehat{\alpha}_{em}^{co} \pm \widehat{\alpha}_{me}^{co} + \frac{1}{\eta_0}\widehat{\alpha}_{mm}^{cr} = 0. \end{aligned} \tag{21}$$

Because in the expressions for the reflected and transmitted fields (18) and (19) the terms proportional to $\overline{\overline{I}}_t$ and $\overline{\overline{J}}_t$ are orthogonal, these conditions are also the necessary conditions for total absorption. The exception for the last statement is the case of circularly polarized incident waves, when these conditions are sufficient but not necessary, opening still more design possibilities if only circularly polarized waves should be absorbed totally. For circularly polarized incidence, the total absorption conditions read

$$\begin{aligned} &\eta_0\widehat{\alpha}_{ee}^{co} \pm \widehat{\alpha}_{em}^{cr} \pm \widehat{\alpha}_{me}^{cr} - \frac{1}{\eta_0}\widehat{\alpha}_{mm}^{co} \\ &\qquad = (\pm j)\left[\eta_0\widehat{\alpha}_{ee}^{cr} \mp \widehat{\alpha}_{em}^{co} \mp \widehat{\alpha}_{me}^{co} - \frac{1}{\eta_0}\widehat{\alpha}_{mm}^{cr}\right] \\ &1 - \frac{j\omega}{2S}\left(\eta_0\widehat{\alpha}_{ee}^{co} \pm \widehat{\alpha}_{em}^{cr} \mp \widehat{\alpha}_{me}^{cr} + \frac{1}{\eta_0}\widehat{\alpha}_{mm}^{co}\right) \\ &\qquad = (\mp j)\frac{j\omega}{2S}\left[\eta_0\widehat{\alpha}_{ee}^{cr} \mp \widehat{\alpha}_{em}^{co} \pm \widehat{\alpha}_{me}^{co} + \frac{1}{\eta_0}\widehat{\alpha}_{mm}^{cr}\right]. \end{aligned} \tag{22}$$

Here $\pm j$ coefficients correspond to the two orthogonal polarizations of the incident circularly polarized fields.

In the following, we will use the general sufficient conditions (21) which ensure total absorption for arbitrary polarization of the incident waves. As one can see, these conditions connect symmetric and antisymmetric parts of the electric and magnetic polarizabilities to the antisymmetric and symmetric parts of the cross-coupling polarizabilities, respectively. This is a very important point because for reciprocal particles (for example, arbitrary shaped metal or dielectric particles) the antisymmetric parts of the electric and magnetic polarizabilities are zero (e.g., [10]) which limits symmetric components of electromagnetic coupling. Furthermore, we see from (21) that for zero reflection it is not necessary to have absorption inside the particles, while it is necessary for zero transmission (note the imaginary quantity in the right-hand side of the third equation).

Let us first analyze layers which exhibit the total absorption property from both sides of the sheet. In this case, conditions (21) should hold for both choices of the $\pm$ signs, and we find that all the magnetoelectric coefficients must vanish:

$$\widehat{\alpha}_{em}^{cr} = \widehat{\alpha}_{me}^{cr} = \widehat{\alpha}_{em}^{co} = \widehat{\alpha}_{me}^{co} = 0. \tag{23}$$

Thus, we conclude that the only possible realization of total absorbers in form of a single layer of particles is the use of electrically and magnetically polarizable uniaxial particles

with the polarizabilities balanced as in a Huygens' pair:

$$\widehat{\alpha}_{ee}^{co} = \frac{S}{j\omega\eta_0}, \qquad \widehat{\alpha}_{mm}^{co} = \eta_0^2 \widehat{\alpha}_{ee}^{co} \tag{24}$$

(and all the other polarizability components equal zero). The effective polarizabilities which include the effect of particle interactions in the array should be, thus, purely imaginary, corresponding to a resonance where the particles show purely absorptive properties.

The relations between the collective polarizabilities and the polarizabilities of the same particles in free space (11) in this special case simplify to

$$\frac{1}{\widehat{\alpha}_{ee}^{co}} = \frac{1}{\alpha_{ee}^{co}} - \beta_e, \qquad \frac{1}{\widehat{\alpha}_{mm}^{co}} = \frac{1}{\alpha_{mm}^{co}} - \beta_m. \tag{25}$$

Using the known expression for the interaction constants in regular dipolar arrays [35, eq. (4.89)], we can find the required particle polarizabilities in free space:

$$\frac{1}{\alpha_{ee}^{co}} = \mathrm{Re}(\beta_e) + j\frac{k^3}{6\pi\epsilon_0} + j\frac{\omega\eta_0}{2S}, \tag{26}$$

$$\frac{1}{\alpha_{mm}^{co}} = \mathrm{Re}(\beta_m) + j\frac{k^3}{6\pi\mu_0} + j\frac{\omega}{2S\eta_0}. \tag{27}$$

We again see that the reactive response of the individual particles should be such that together with the reactive part of the interaction field a resonance condition is satisfied. We can also check that the amplitude of the secondary plane waves created by the two dipolar arrays of the perfect absorber equal to one half of the incident field amplitude:

$$E_{\rm sc} = -\frac{\eta_0}{2}\frac{j\omega p}{S} = -\frac{\eta_0}{2}\frac{j\omega}{S}\widehat{\alpha}_{ee}^{co}E_{\rm inc} = -\frac{1}{2}E_{\rm inc} \tag{28}$$

(we have substituted $\widehat{\alpha}_{ee}^{co}$ from (24)). The field created by the magnetic-dipole array has the same amplitude. In the forward directions, the sum of these two plane waves compensates the incident field, and in the reflection direction these two plane waves are out of phase and the sum is zero.

2) Total absorption from one side of the sheet: Next, we consider single-layer sheets which work as total absorbers only from one side and study what functionalities can be engineered for illumination from the opposite side. From (21), we know that the presence of cross-coupling polarizabilities (as well as the anti-symmetric parts of the electric and magnetic polarizabilities) causes asymmetry in interactions of the sheet with incident waves coming from the opposite directions. Let us assume that we satisfy (21) for waves incident from one of the two sides. This corresponds to conditions

$$\begin{aligned}
\eta_0\widehat{\alpha}_{ee}^{co} - \frac{1}{\eta_0}\widehat{\alpha}_{mm}^{co} &= \mp(\widehat{\alpha}_{em}^{cr} + \widehat{\alpha}_{me}^{cr}) \\
\eta_0\widehat{\alpha}_{ee}^{cr} - \frac{1}{\eta_0}\widehat{\alpha}_{mm}^{cr} &= \pm(\widehat{\alpha}_{em}^{co} + \widehat{\alpha}_{me}^{co}) \\
\eta_0\widehat{\alpha}_{ee}^{co} + \frac{1}{\eta_0}\widehat{\alpha}_{mm}^{co} &= \frac{2S}{j\omega} \mp (\widehat{\alpha}_{em}^{cr} - \widehat{\alpha}_{me}^{cr}) \\
\eta_0\widehat{\alpha}_{ee}^{cr} + \frac{1}{\eta_0}\widehat{\alpha}_{mm}^{cr} &= \pm(\widehat{\alpha}_{em}^{co} - \widehat{\alpha}_{me}^{co}).
\end{aligned} \tag{29}$$

Here, as above, the upper sign corresponds to $-\mathbf{z}_0$ directed and the lower sign to the oppositely-directed incident plane waves. Using the same conditions for total absorption for the opposite incidence direction (taking the lower signs in (21), (18), and (19)), we find the reflected and transmitted electric fields for the same sheet when the incidence is from the other side:

$$\mathbf{E}_r = \frac{j\omega}{S}\left\{\pm[\widehat{\alpha}_{em}^{cr} + \widehat{\alpha}_{me}^{cr}]\,\overline{\overline{I}}_t \mp [\widehat{\alpha}_{em}^{co} + \widehat{\alpha}_{me}^{co}]\,\overline{\overline{J}}_t\right\}\cdot\mathbf{E}_{\rm inc} \tag{30}$$

$$\mathbf{E}_t = \frac{j\omega}{S}\left\{\pm[\widehat{\alpha}_{em}^{cr} - \widehat{\alpha}_{me}^{cr}]\,\overline{\overline{I}}_t \mp [\widehat{\alpha}_{em}^{co} - \widehat{\alpha}_{me}^{co}]\,\overline{\overline{J}}_t\right\}\cdot\mathbf{E}_{\rm inc}. \tag{31}$$

These equations show that tuning the layer to act as a perfect absorber from one side, it is possible to realize some special properties (in reflection and transmission) from the other side.

To study these properties, we begin with the case of reciprocal structures. In this case, the electric and magnetic polarizabilties are symmetric dyadics ($\widehat{\alpha}_{ee}^{cr} = 0$, $\widehat{\alpha}_{mm}^{cr} = 0$) and the fields coupling coefficients satisfy

$$\widehat{\alpha}_{em}^{co} = -\widehat{\alpha}_{me}^{co}, \qquad \widehat{\alpha}_{em}^{cr} = \widehat{\alpha}_{me}^{cr}, \tag{32}$$

corresponding to chiral and omega couplings [10]. Due to the reciprocity, the transmission coefficient is zero for waves incident from both sides. The second equation in (29) is satisfied identically, and from the last one, we see that the chirality parameter $\widehat{\alpha}_{me}^{co} = 0$. Thus, if the sheet is tuned to work as a perfect absorber from one side, the chirality parameter is zero and there is no possibility to tune the reflection properties from the opposite side introducing chirality.

On the other hand, the omega coupling coefficient $\widehat{\alpha}_{me}^{cr}$ is not fixed by the total absorption condition on one side, because from the first and third equations in (29) we find

$$\widehat{\alpha}_{ee}^{co} = \frac{S}{j\omega\eta_0} \mp \frac{1}{\eta_0}\widehat{\alpha}_{me}^{cr} \tag{33}$$

$$\widehat{\alpha}_{mm}^{co} = \frac{\eta_0 S}{j\omega} \pm \eta_0\widehat{\alpha}_{me}^{cr}. \tag{34}$$

Comparing with (24), we see that introducing omega coupling, we can maintain the property of total absorption from one of the sides with relaxed requirements on the electric and magnetic polarizabilities. For instance, we can possibly engineer the omega coupling parameter $\widehat{\alpha}_{me}^{cr}$ so that the required magnetic polarizability is much smaller than that dictated by (24). The reflection coefficient from the side opposite to the matched one we find from (30):

$$\mathbf{E}_r = \pm\frac{2j\omega}{S}\widehat{\alpha}_{em}^{cr}\,\overline{\overline{I}}_t\cdot\mathbf{E}_{\rm inc}. \tag{35}$$

Thus, varying the omega coupling parameter, we can control the co-polarized reflection from the opposite side of the sheet, maintaining the matching and total absorption properties from one side.

More functionalities become available if we allow nonreciprocal response of the particles. For simplicity, let us concentrate here on the cases where the magnetoelectric coupling

is only due to nonreciprocity, assuming that the chirality and omega coupling coefficients are zero (the effects of chirality and omega coupling have been considered above). In these cases, the coupling coefficients satisfy

$$\widehat{\alpha}_{em}^{co}=\widehat{\alpha}_{me}^{co},\qquad \widehat{\alpha}_{em}^{cr}=-\widehat{\alpha}_{me}^{cr}, \tag{36}$$

corresponding to Tellegen and "moving" particles, respectively [10]. From the first equation in (29), we find that $\eta_0\widehat{\alpha}_{ee}^{co}=\frac{1}{\eta_0}\widehat{\alpha}_{mm}^{co}$ (Huygens' relation). From this and the third relation we get

$$\widehat{\alpha}_{ee}^{co}=\frac{S}{j\omega\eta_0}\mp\frac{1}{\eta_0}\widehat{\alpha}_{em}^{cr}. \tag{37}$$

The second and the last relations in (29) connect the antisymmetric parts of the electric and magnetic polarizabilities with the Tellegen parameter:

$$\widehat{\alpha}_{ee}^{cr}=\pm\frac{1}{\eta_0}\widehat{\alpha}_{em}^{co},\qquad \widehat{\alpha}_{mm}^{cr}=\mp\eta_0\widehat{\alpha}_{em}^{co}. \tag{38}$$

Thus, if the Tellegen coupling is present, its effects should be balanced with the nonreciprocity in both electric and magnetic polarizabilities. Tellegen coupling allows control of the reflection coefficient from the opposite side, since

$$\mathbf{E}_r=\mp\frac{2j\omega}{S}\widehat{\alpha}_{em}^{co}\overline{\overline{J}}_t\cdot\mathbf{E}_{\rm inc}. \tag{39}$$

We see that the Tellegen sheet can be designed to work as a perfect absorber from one side and a twist polarizer in reflection from the other side.

Finally, the antisymmetric part of the nonreciprocal coupling coefficient allows to control the transmission coefficient from the opposite side (for nonreciprocal sheets the transmission coefficient is not anymore necessarily symmetric):

$$\mathbf{E}_t=\pm\frac{2j\omega}{S}\widehat{\alpha}_{em}^{cr}\overline{\overline{I}}_t\cdot\mathbf{E}_{\rm inc}. \tag{40}$$

This transmission coefficient equals unity if $\widehat{\alpha}_{em}^{cr}=\pm S/(2j\omega)$, in which case equation (37) shows that all the polarizabilities are in balance

$$\eta_0\widehat{\alpha}_{ee}^{co}=\pm\widehat{\alpha}_{em}^{cr}=\frac{1}{\eta_0}\widehat{\alpha}_{mm}^{co}=\frac{S}{j2\omega}. \tag{41}$$

Using (11), it is easy to show that the polarizabilities for each individual particle should also be in balance and equal to

$$\eta_0\alpha_{ee}^{co}=\pm\alpha_{em}^{cr}=\frac{1}{\eta_0}\alpha_{mm}^{co}=\frac{\eta_0}{2}\frac{1}{\frac{j\omega\eta_0}{S}+\beta_e}. \tag{42}$$

We see that the required electric and magnetic effective polarizabilities are twice as small as compared to the simple case of isotropic dipole particles (24). However, the resulting amplitudes of the induced dipole moments and the amplitudes of the secondary plane waves are the same, because both moments are generated by both incident fields. If this nonreciprocal array is excited from the absorbing side, these secondary plane waves cancel the incident wave behind the sheet and they cancel each other in the reflection direction, same as for the simple isotropic array. But for the excitation from the opposite side, the induced dipole moments are zero, because contributions due to the applied electric and magnetic fields cancel out, and the sheet is transparent. We can conclude that this interesting structure has the property of the ultimately thin (a single layer of dipole particles) isolator: from one side it acts as a total absorber while from the other side, the sheet is transparent. Moreover, it appears that this is the only possible configuration having this property.

III. Uniaxial bi-anisotropic particles as components of totally absorbing arrays

Next we will discuss some possible designs of bi-anisotropic particles with the properties required for single-layer perfect absorbers. From the reciprocal classes, the most interesting and practically useful property is the omega coupling, since this effect gives flexibility in the requirements on the electric and magnetic polarizabilities and allows control over the reflection coefficient from the back side of the absorbing sheet (see (33)–(35)).

A. Wire omega particles

The classical topology of bi-anisotropic particles with omega coupling is an Ω-shaped particle [10], [30], [31]. A single omega particle made of a conducting wire (see picture in Table 1), in approximation of electrically small particles, in the microwave frequency range can be modeled by the following polarizabilities [10]:

$$\begin{aligned}\alpha_{ee}^{co}&=\frac{l^2}{j\omega(Z_l+Z_w)}\qquad \alpha_{mm}^{co}=-\mu_0^2\frac{j\omega(\pi r^2)^2}{Z_l+Z_w}\\ \alpha_{em}^{cr}&=\mp\frac{\mu_0\pi r^2 l}{Z_l+Z_w}\qquad \alpha_{me}^{cr}=\mp\frac{\mu_0\pi r^2 l}{Z_l+Z_w},\end{aligned} \tag{43}$$

where r is the radius of the loop, r_0 is the wire radius, $2l$ is the dipole length, Z_w and Z_l are the input impedances for the wire and loop antennas as the two connected parts of the particle.

It is clear from (43) that

$$\alpha_{ee}^{co}\alpha_{mm}^{co}=-\alpha_{em}^{cr}\alpha_{me}^{cr}=-(\alpha_{em}^{cr})^2. \tag{44}$$

This condition is a limitation on electromagnetic properties of a wire omega particle. Using (11), we find that the effective polarizabilities of omega particles forming a periodical array satisfy the same relation

$$\widehat{\alpha}_{ee}^{co}\widehat{\alpha}_{mm}^{co}=-(\widehat{\alpha}_{em}^{cr})^2. \tag{45}$$

Let us consider the limitation (45) together with the first condition for total absorption in (29) (which is the condition for zero reflection from an array of omega particles). Combining these two equations, we get

$$\widehat{\alpha}_{ee}^{co}\pm\frac{2j}{\eta_0}\sqrt{\widehat{\alpha}_{ee}^{co}\widehat{\alpha}_{mm}^{co}}-\frac{1}{\eta_0^2}\widehat{\alpha}_{mm}^{co}=0. \tag{46}$$

From this simple quadratic equation one can obtain the relation

$$\widehat{\alpha}_{ee}^{co}=-\frac{1}{\eta_0^2}\widehat{\alpha}_{mm}^{co} \tag{47}$$

and using (11), we get the same relation between the polarizabilities of individual particles in free space

$\left(\alpha_{ee}^{co}=-\frac{1}{\eta_0^2}\alpha_{mm}^{co}\right)$, but this equation does not hold for passive omega particles, because the different signs of the imaginary parts of the electric and magnetic polarizabilities mean that the particle should be active. On the other hand, it is impossible to satisfy the third condition from (34) taking the limitation of (47) into account. Therefore, a wire omega particles cannot be used for the design of perfect absorbers of this type. This is an interesting fact because earlier nearly-total absorption was predicted in structures which behave like omega particles [32]. However, there is a significant difference between the case studied in [32] and wire omega particles. For a wire omega particle all the polarizabilities have the same resonance frequency, but for the structure in [32], one can tune the structural parameters so that different polarizabilities have different resonance frequencies. Thus, it appears possible to break the limitation of (47) using other kinds of omega particles and achieve total absorption with the help of the omega-coupling phenomenon.

B. Omega-Tellegen particles

Within the nonreciprocal classes, the most interesting properties are the possibilities offered by nonreciprocal field coupling phenomena in array of particles. Realization of such particles requires inclusions of some nonreciprocal elements. The known structures for the microwave frequency range [10] include magnetized ferrite spheres coupled to specially shaped metal elements, see illustrations in Table 1. However, both these structures exhibit also reciprocal field coupling effects in addition to the desired nonreciprocal effects.

A single uniaxial Tellegen particle shows also some omega field coupling due to the asymmetrical position of the metal strips with respect to the center of the ferrite sphere. For this reason, we call it omega-Tellegen particle. Its polarizability dyadics have the form

$$\begin{cases} \overline{\overline{\widehat{\alpha}}}_{ee}=\widehat{\alpha}_{ee}^{co}\overline{\overline{I}}_t+\widehat{\alpha}_{ee}^{cr}\overline{\overline{J}}_t \\ \overline{\overline{\widehat{\alpha}}}_{mm}=\widehat{\alpha}_{mm}^{co}\overline{\overline{I}}_t+\widehat{\alpha}_{mm}^{cr}\overline{\overline{J}}_t \\ \overline{\overline{\widehat{\alpha}}}_{em}=\overline{\overline{\widehat{\alpha}}}_{me}=\widehat{\alpha}_{em}^{co}\overline{\overline{I}}_t+\widehat{\alpha}_{em}^{cr}\overline{\overline{J}}_t. \end{cases} \tag{48}$$

Using relations (21) and (48), we get the following conditions for total absorption in omega-Tellegen arrays

$$\begin{aligned} \eta_0\widehat{\alpha}_{ee}^{co}\pm 2\widehat{\alpha}_{em}^{cr}-\frac{1}{\eta_0}\widehat{\alpha}_{mm}^{co}&=0\\ \eta_0\widehat{\alpha}_{ee}^{cr}\mp 2\widehat{\alpha}_{em}^{co}-\frac{1}{\eta_0}\widehat{\alpha}_{mm}^{cr}&=0\\ \eta_0\widehat{\alpha}_{ee}^{co}+\frac{1}{\eta_0}\widehat{\alpha}_{mm}^{co}&=\frac{2S}{j\omega}\\ \eta_0\widehat{\alpha}_{ee}^{cr}+\frac{1}{\eta_0}\widehat{\alpha}_{mm}^{cr}&=0. \end{aligned} \tag{49}$$

This shows that if we want to use the advantages offered by Tellegen coupling, we need to design the particle so that the omega coupling coefficient α_{em}^{cr} is properly balanced with the electric and magnetic polarizabilities.

C. Chiral-moving particles

Likewise, the known artificial moving particle [10], [33], [34] (picture in Table 1) exhibits reciprocal magnetoelectric coupling because of its chiral shape. The properties of such particle can be modeled by the polarizability dyadics of the form

$$\begin{cases} \overline{\overline{\widehat{\alpha}}}_{ee}=\widehat{\alpha}_{ee}^{co}\overline{\overline{I}}_t+\widehat{\alpha}_{ee}^{cr}\overline{\overline{J}}_t \\ \overline{\overline{\widehat{\alpha}}}_{mm}=\widehat{\alpha}_{mm}^{co}\overline{\overline{I}}_t+\widehat{\alpha}_{mm}^{cr}\overline{\overline{J}}_t \\ \overline{\overline{\widehat{\alpha}}}_{em}=-\overline{\overline{\widehat{\alpha}}}_{me}=\widehat{\alpha}_{em}^{co}\overline{\overline{I}}_t+\widehat{\alpha}_{em}^{cr}\overline{\overline{J}}_t. \end{cases} \tag{50}$$

Using the relations (21) and (50), the conditions for total absorption in the chiral-moving slab read

$$\begin{aligned} \eta_0\widehat{\alpha}_{ee}^{co}-\frac{1}{\eta_0}\widehat{\alpha}_{mm}^{co}&=0\\ \eta_0\widehat{\alpha}_{ee}^{cr}-\frac{1}{\eta_0}\widehat{\alpha}_{mm}^{cr}&=0\\ \eta_0\widehat{\alpha}_{ee}^{co}\pm 2\widehat{\alpha}_{em}^{cr}+\frac{1}{\eta_0}\widehat{\alpha}_{mm}^{co}&=\frac{2S}{j\omega}\\ \eta_0\widehat{\alpha}_{ee}^{cr}\mp 2\widehat{\alpha}_{em}^{co}+\frac{1}{\eta_0}\widehat{\alpha}_{mm}^{cr}&=0. \end{aligned} \tag{51}$$

In this case, the chirality parameter α_{em}^{co} should be balanced with the anti-symmetric parts of the electric and magnetic polarizabilities. Implementation of total-absorption arrays using omega-Tellegen or chiral-moving particles presents significant difficulties. To the best of our knowledge, there are no analytical models to calculate the individual polarizabilities of these particles. The relations between the effective and individual polarizabilities for these particles become more involved. Finally, the design presupposes the use of ferrites and magnetic field bias which presents practical difficulties. On the other hand, these topologies offer unique properties such as a thin sheet operating as an isolator, and they clearly deserve further studies.

IV. Conclusions

We have considered possible approaches for realization of perfect absorbers using ultimately thin (single layers of particles) structures. The thickness cannot be strictly zero (in the electromagnetic sense) because we must allow magnetic response in the layer. We have demonstrated that to realize total absorption from both sides of the sheet, we just need to realize balanced electric and magnetic polarizabilities and all magnetoelectric polarizabilities should be zero. Further, we considered single-layer sheets which operate as perfect absorbers only when illuminated from one of the two sides of the sheet and studied what functionalities can be engineered for illumination from the opposite side. We have shown that introducing omega coupling in the constituent particles makes it possible to realize a layer which acts as a perfect absorber from one side with controllable co-polarized reflection from the opposite side of the sheet. For reciprocal structures, it has been shown that tuning the layer to act as a perfect absorber from one side does not allow to have any chirality in the layer. We have seen that allowing nonreciprocity in the properties of the absorbing particles offers possibilities

for more functionalities. A Tellegen sheet can be designed to work as a perfect absorber from one side and a twist polarizer in reflection from the other side. The antisymmetric part of the nonreciprocal coupling coefficient (i.e., a layer of particles with the constitutive parameters of an artificial moving medium) makes it possible to achieve total absorption from one side, but controlled transmission coefficient from the opposite side. In particular, the regime when the layer acts as a perfect absorber from one side, while from the other side the sheet is transparent, is possible. This corresponds to the ultimately thin isolator. Finally, we have studied some particular examples of possible realizations of single-layer perfect absorbers with the use of omega, omega-Tellegen, and chiral-moving particles as canonical examples of uniaxial bi-anisotropic particles.

REFERENCES

[1] E. F. Knott, J. F. Shaeffer, and M. T. Tuley, "Radar absorbing materials," in *Radar cross section*, $2^{\mathbf{nd}}$ ed., Boston: Artech House, 1993, pp. 297-360.

[2] B. A. Munk, "Jaumannand Circuit Analog Absorbers," in *Frequency-selective surfaces: Theory and design*, New York: Wiley, 2000, pp. 315-335.

[3] F. Mayer, T. Ellam, and Z. Cohn, "High frequency broadband absorption structures," in *IEEE International Symposium on Electromagnetic Compatibility*, vol. 2, 1998, pp. 894-899.

[4] F. Bilotti, A. Alu, N. Engheta, and L. Vegni, "Compact microwave absorbers utilizing single negative metamaterial layers," in *Proc. of Joint IEEE AP-S/URSI Symposium*, Albuquerque, USA, 2006.

[5] F. Bilotti, A. Toscano, K. B. Alici, E. Ozbay, and L. Vegni, "Design of miniaturized narrowband absorbers based on resonant-magnetic inclusions," *Electromagnetic Compatibility, IEEE Transactions on*, vol. 53, no. 1, pp. 63-72, Feb. 2011.

[6] D. Pozar, "Scattered and absorbed powers in receiving antennas," *Antennas and Propagation Magazine, IEEE*, vol. 46, no. 1, pp. 144-145, Feb. 2004.

[7] C. L. Holloway, E. F. Kuester, J. A. Gordon, J. O'Hara, J. Booth, and D. R. Smith, "An overview of the theory and applications of metasurfaces: The two-dimensional equivalents of metamaterials," *Antennas and Propagation Magazine, IEEE*, vol. 54, no. 2, pp. 10-35, Apr. 2012.

[8] Do-Hoon Kwon and D. M. Pozar, "Analysis of maximum received power by arbitrary lossless arrays," *Antennas and Propagation Society International Symposium, APSURSI, IEEE*, Jun. 2009, pp. 1-4.

[9] L. B. Felsen and N. Marcuvitz, *Radiation and scattering of waves*, Piscataway, NJ: IEEE Press, 1994.

[10] A. N. Serdyukov, I. V. Semchenko, S. A. Tretyakov, and A. Sihvola, *Electromagnetics of bi-anisotropic materials: Theory and applications*, Amsterdam: Gordon and Breach Science Publishers, 2001.

[11] A. Solovey and R. Mittra, "Theoretical limitations on shielding and reflective properties of microwave absorbers," *European Microwave Conference*, 2009, pp. 1614-1617.

[12] V. K. Varadan, V. V. Varadan, and A. Lakhtakia, "On the possibility of designing anti-reflection coating using chiral composites," *J. Wave-Mater. Interact.*, vol. 2, no. 1, pp. 71-81, 1987.

[13] D. L. Jaggard, N. Engheta, "Chirosorb^{TM} as an invisible medium," *Electronics Letters*, vol. 25, no. 3, pp. 173-174, Feb. 1989.

[14] J. C. Monzon, D. L. Jaggard, and N. Engheta, "Comment on chirosorb as an invisible medium (and reply)," *Electronics Letters*, vol. 25, no. 16, pp. 1060-1061, Aug. 1989.

[15] C. F. Bohren, R. Luebbers, H. S. Langdon, and F. Hunsberger, "Microwave-absorbing chiral composites: Is chirality essential or accidental?," *Applied Optics*, vol. 31, pp. 6403-6407, Oct. 1992.

[16] S. A. Tretyakov, A. A. Sochava, C. R. Simovski, "Influence of chiral shapes of individual inclusions on the absorption in chiral composite coatings," *Electromagnetics*, vol. 16, pp. 113-127, 1996.

[17] J. H. Cloete, M. Bingle, D. B. Davidson, "The role of chirality in synthetic microwave absorbers," in *Proc. of Electroomagn. in Advance Applications (ICEAA)*, Torino, Italy, 1999, pp. 55-58; M. Bingle, "The role of chirality in synthetic microwave absorbers," PhD Dissertation, University of Stellenbosch, South Africa, 1998.

[18] B. Wang, T. Koschny, and C. M. Soukoulis, "Wide-angle and polarization-independent chiral metamaterial absorber," *Phys. Rev. B*, vol. 28, no. 033108, 2009.

[19] S. A. Tretyakov and A. A. Sochava, "Novel uniaxial bianisotropic materials: reflection and transmission in planar structures," *Progress In Electromagnetics Research, PIER 9*, pp. 157-179, 1994.

[20] T. G. Kharina, S. A. Tretyakov, A. A. Sochava, C. R. Simovski, and S. Bolioli, "Experimental studies of artificial omega media," *Electromagnetics*, vol. 18, no. 4, pp. 423-437, 1998.

[21] N. I. Landy, S. Sajuyigbe, J. J. Mock, D. R. Smith, and W. J. Padilla, "Perfect Metamaterial Absorber," *Phys. Rev. Lett.*, vol. 100, no. 207402, 2008.

[22] J. Hao, J. Wang, X. Liu, W. J. Padilla, L. Zhou, and M. Qiu, "High performance optical absorber based on a plasmonic metamaterial ," *Appl. Phys. Lett.*, vol. 96, no. 251104,2010.

[23] P. K. Singh, K. A. Korolev, M. N. Afsar, and S. Sonkusale, "Single and dual band 77/95/110GHz metamaterial absorbers on flexible polyimide substrate ," *Appl. Phys. Lett.*, vol. 99, no. 264101, 2011.

[24] H. Li, L. H. Yuan, B. Zhou, X. P. Shen, Q. Cheng, and T. J. Cui, "Ultrathin multiband gigahertz metamaterial absorbers," *J. Appl. Phys.*, vol. 100, no. 014909, 2011.

[25] X. Shen, T. J. Cui, J. Zhao, H. F. Ma, W. X. Jiang, and H. Li, "Polarization-independent wide-angle triple-band metamaterial absorber," *Optics Express*, vol. 19, no. 4, pp. 9401-9407, 2011.

[26] F. Ding, Y. Cui, X. Ge, Y. Jin, and S. He, "Ultra-broadband microwave metamaterial absorber," *Appl. Phys. Lett.*, vol. 100, no. 103506, 2012.

[27] C. Wu and G. Shvets, "Design of metamaterial surfaces with broadband absorbance," *Optics Letters*, vol. 37, no. 3, 2012.

[28] S. A. Tretyakov, A. H. Sihvola, A. A. Sochava, and C. R. Simovski, "Magnetoelectric interactions in bi-anisotropic media," *Journal of Electromagnetic Waves and Applications*, vol. 12, no. 4, pp. 481-497, 1998.

[29] T. Niemi, A. O. Karilainen, and S. A. Tretyakov, "Synthesizing a twist polarizer," *IEEE International Symposium on Antennas and Propagation and USNC/URSI National Radio Science Meeting (AP-S/URSI)*, paper IF54p10, Chicago, USA, 2012.

[30] M. M. I. Saadoun and N. Engheta, "A reciprocal phase shifter using novel pseudochiral or Ω medium," *Microwave and Optical Technology Letters*, vol. 5, pp. 184-188, Apr. 1992.

[31] S. A. Tretyakov, A. A. Sochava, "Proposed composite material for nonreflecting shields and antenna radomes," *Electronics Letters*, vol. 29, no. 12, pp. 1048-1049, Jun. 1993.

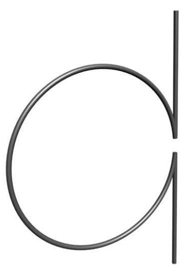
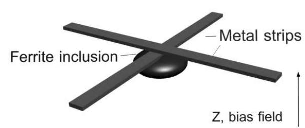
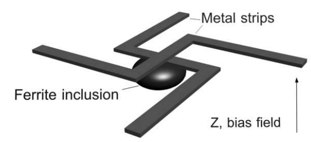
[32] M. Albooyeh and C. R. Simovski, "Huge local field enhancement in perfect plasmonic absorbers," *Optics Express*, vol. 20, pp. 21888-21895, Sep. 2012.

[33] S.A. Tretyakov, "Physical interpretation of the transparent absorbing boundary for the truncation of the computational domain," *IEEE Microwave and Guided Wave Letters*, vol. 8, no. 10, pp. 321-323, Oct. 1998.

[34] S.A. Tretyakov, "Nonreciprocal composite with the material relations of the transparent absorbing boundary," *Microwave and Optical Technology Letters*, vol. 19, pp. 365-368, Dec. 1998.

[35] S. Tretyakov, *Analytical Modeling in Applied Electromagnetics*, Norwood, MA: Artech House, 2003.

TABLE I: Conditions for perfect absorption

Condition for total absorption		
Wire Omega	Omega—Tellegen	Chiral—Moving
		
$\widehat{\alpha}_{ee}^{co}=-\frac{1}{\eta_0^2}\widehat{\alpha}_{mm}^{co}$	$\eta_0\widehat{\alpha}_{ee}^{co}\pm 2\widehat{\alpha}_{em}^{cr}-\frac{1}{\eta_0}\widehat{\alpha}_{mm}^{co}=0$ $\eta_0\widehat{\alpha}_{ee}^{cr}\mp 2\widehat{\alpha}_{em}^{co}-\frac{1}{\eta_0}\widehat{\alpha}_{mm}^{cr}=0$ $\eta_0\widehat{\alpha}_{ee}^{co}+\frac{1}{\eta_0}\widehat{\alpha}_{mm}^{co}=\frac{2S}{j\omega}$ $\eta_0\widehat{\alpha}_{ee}^{cr}+\frac{1}{\eta_0}\widehat{\alpha}_{mm}^{cr}=0$	$\eta_0\widehat{\alpha}_{ee}^{co}-\frac{1}{\eta_0}\widehat{\alpha}_{mm}^{co}=0$ $\eta_0\widehat{\alpha}_{ee}^{cr}-\frac{1}{\eta_0}\widehat{\alpha}_{mm}^{cr}=0$ $\eta_0\widehat{\alpha}_{ee}^{co}\pm 2\widehat{\alpha}_{em}^{cr}+\frac{1}{\eta_0}\widehat{\alpha}_{mm}^{co}=\frac{2S}{j\omega}$ $\eta_0\widehat{\alpha}_{ee}^{cr}\mp 2\widehat{\alpha}_{em}^{co}+\frac{1}{\eta_0}\widehat{\alpha}_{mm}^{cr}=0$
Reflected and transmitted fields from the other side of a single-sided perfect absorber		
Omega	Tellegen	Moving
$\mathbf{E}_r=\pm\frac{2j\omega}{S}\widehat{\alpha}_{em}^{cr}\overline{\overline{I}}_t\cdot\mathbf{E}_{\rm inc}$ $\mathbf{E}_t=0$	$\mathbf{E}_r=\mp\frac{2j\omega}{S}\widehat{\alpha}_{em}^{co}\overline{\overline{J}}_t\cdot\mathbf{E}_{\rm inc}$ $\mathbf{E}_t=0$	$\mathbf{E}_r=0$ $\mathbf{E}_t=\pm\frac{2j\omega}{S}\widehat{\alpha}_{em}^{cr}\overline{\overline{I}}_t\cdot\mathbf{E}_{\rm inc}$